Reinventing the Wheel: Historical Perspectives on Theories for Interpreting Discourse Patterns in Mathematics Classrooms

Nerida F. Ellerton
Illinois State University
<ellerton@ilstu.edu>

Pongchawee Vaiyavutjamai
Chiang Mai University
<edipvyvt@chiangmai.ac.th>

M. A. (Ken) Clements
Illinois State University
<clements@ilstu.edu>

Many key aspects of modern theory relating to classroom discourse in mathematics education, especially the theory of didactical situations, have not progressed much further than theories put forward in the 1840s by the North American educator David Page. Aspects of Brousseau's (1997) description of what he called the Topaze effect, are remarkably similar to Page's idea of "drawing out"—a mode of questioning that was likely to remove the cognitive challenge for learners engaged in non-trivial mathematical tasks.

19th-Century Theories on Discourse Factors Influencing Mathematics Learning

Johann Heinrich Pestalozzi on the Development of Concepts

During 1820–1880 the ideas of Johann Heinrich Pestalozzi (1746–1827), a Swiss-German educator, became well known in Western Europe and the United States. Pestalozzi's central thesis was that although a child's intellectual, physical, and moral development (the "head", the "hand", and the "heart") unfolded according to well-defined rules, rather like a plant in the ground, she or he learned best inductively, through meaningful examples. Pestalozzi insisted that the main task of the teacher was to nurture the growing organism so that it could reach its full potential. Attempts to force a child's learning along unnatural paths would be counter-productive (Soëtard, 1981, 1994).

Thus, Pestalozzi maintained, education practices should aim at developing mental and physical strengths through sequential exercises done systematically and in a prescribed order, first at home and then at school. Pestalozzi's theory of elementary education included intellectual, moral, physical, and vocational training, all closely interacting to produce harmonious human development. For Pestalozzi, transmission of concepts, skills and principles from a teacher to a student through words alone was impossible. Teachers needed to introduce students to appropriately-structured sense impressions of what they wanted their students to learn (Soëtard, 1981, 1994). But that did not imply that teachers should not ask questions or make statements in order to help learners advance their thinking.

One of the weaknesses of many interpreters of Pestalozzian writing has been an all-or-nothing mentality. Pestalozzi made it clear that he believed that after children had gained an idea of the objects that were now part of their environments, teachers should name the objects and get students to discuss and compare various aspects of them. This would permit abstraction, so that definitions of components of the objects would become meaningful to the learners. Petalozzi's ideas can be thought of as developing the constructivist tradition in education, from John Locke, in England, through Jean Jacques Rousseau, in France.

In J. Dindyal, L. P. Cheng & S. F. Ng (Eds.), Mathematics education: Expanding horizons (Proceedings of the 35th annual conference of the Mathematics Education Research Group of Australasia). Singapore: MERGA.

Warren Colburn—Pestalozzi's Disciple in the United States

One of Pestalozzi's chief disciples in the United States was Warren Colburn (1783–1833), America's first major theoretician in the field of mathematics education (Bidwell & Clason, 1970). Colburn's (1821) first publication, *Arithmetic on the Plan of Pestalozzi* took the U.S. education world by storm. In 1822 a new edition, with a different title—*First Lessons in Arithmetic on the Plan of Pestalozzi*—was published, and so was *Arithmetic, Being a Sequel to the First Lessons in Arithmetic* (Colburn, 1822*a*, 1822*b*).

Colburn reiterated Pestalozzi's call for instruction in early arithmetic to be linked with the learners' senses. He rejected the prevailing idea that mainly boys should learn arithmetic, and that schools should not attempt to teach mathematical concepts to children less than 10 years of age. For Colburn, as for Pestalozzi, all children—boys and girls—should learn arithmetic from the age of six. The emphasis should be on developing classroom discourse patterns by which learners constructed concepts inductively. Teacher-student discourse should guide students toward the construction of appropriate concepts. Although the teacher's role was not to tell, nevertheless the right questions needed to be asked at the right time.

David Perkins Page's Theorisation of the "Drawing-Out Process"

The theories on classroom discourse of David Perkins Page (1810–1848), probably the most influential North American education theorist of the 19th century, are hardly known today. Page was a young, but nevertheless experienced high-school teacher who, in 1844, became Principal of the State Normal School in Albany, New York. Over the next three years he was outstandingly successful in that capacity, but after a brief illness he died in January 1848, aged just 38. He wrote just one book. *Theory and Practice of Teaching,* but that book became the major 19th century theoretical treatise on teaching and learning for North America and East Asian schools and colleges. What is pertinent for this paper is that Page specifically theorised what we shall call the concept of "cognitive emptying".

Page (1850) powerfully drew attention to how many teachers thought that they were teaching using an inductive form of pedagogy when, in fact, they had developed a very damaging "cognitive-emptying" approach which he referred to as the "drawing-out process" (p. 79). He stated that he had seen this process done so adroitly by a teacher that "a company of visitors would agree that it was wonderful to see how thoroughly the children had been instructed!" (p. 81). This "drawing-out process" involved what lawyers called *leading questions.* It was practised, usually, whenever over-zealous teachers desired to help along their pupils. It could occur when a teacher was questioning an individual student (as in a "recitation" session) or in a session when the teacher was addressing a whole class of students. We illustrate the concept by reproducing two excerpts from Page (1850).

Page's (1850) first example of the "drawing out" or "cognitive-emptying" process.
Teacher: John, what is the number to be divided called?"
John: Um… Um. …
Teacher: Is it the *dividend*?
John: Yes, Sir. The dividend.
Teacher: Well, what is that which is left after dividing called? –The remainder, is it?"
John: Yes
Teacher: Well, John, of what denomination is the remainder?"
Teacher: Isn't it always the same as the dividend, John?
John: Yes, Sir.

Teacher: Very well, John, what denomination is this dividend? [*points to the work upon the board*] Dollars, is it not?
John: Yes, Sir, dollars.
Teacher: Very well; now what is the remainder? Why dollars, too, isn't it?
John: Oh yes sir, dollars! (pp. 79–80)

Page's (1850) second example of the "drawing out" or "cognitive-emptying" process. A teacher whose school Colburn visited called upon the class to demonstrate what they had studied from a section in Warren Colburn's *First Lessons*. The teacher began by asking where the class should begin in the textbook. Then, the following discourse occurred:

Pupils: On the 80th page, Question 3.
Teacher: Read it, Charles.
Charles: [*Reads*] "A man being asked how many sheep he had, said that he had them in two pastures; in one pasture he had eight; that three-fourth of these were just one-third of what he had in the other. How many were there in the other?
Teacher: Well, Charles, you must get one-fourth of eight, must you not?
Charles: Yes, Sir.
Teacher: Well, one-fourth of eight is two, isn't it?
Charles: Yes sir, one-fourth of eight is two.
Teacher: Well, three-fourth will be three times two, won't it?
Charles: Yes.
Teacher: Well, three times two are six, eh?
Charles*:* Yes Sir.
Teacher: Very well. [*a pause …*] Now the book says that six is one-third of what he had in the other pasture. Doesn't it?
Charles: Yes Sir.
Teacher: Then, if six is one-third, three thirds will be three times six—won't it?
Charles: Yes Sir.
Teacher: And three times six are—eighteen, ain't it?
Charles: Yes Sir!
Teacher: Then he had eighteen sheep in the other pasture, had he?
Charles. Yes Sir.

At this stage of the lesson Page asked if he could request Charles to go through it alone. "Oh yes", said the teacher. "Charles, you may do it again". Charles again read the question and looked up. "Well", said the teacher, "You must first get one-fourth of eight, mustn't you?" "Yes sir". "And, one-fourth of eight is two, isn't it?" "Yes Sir". And so the process went on as before until the final 18 sheep were drawn out, as before. Now the teacher looked around with an air which seemed to say, "Now I suppose you are satisfied".

"Shall I ask Charles to do it again?" said I. The teacher assented. Charles again read the question—and looked up. I waited, and he waited—but the teacher could *not* wait. "Why, Charles", said he impatiently, "You want one-fourth of eight, don't you? "Yes sir", said Charles, promptly. And I thought best not to insist further" (pp. 82–83). Page admitted that this was an extreme case, but said it was a fair sample of the discourse patterns in that teacher's class during arithmetic lessons. According to Page, this habit of assisting the pupil by asking and answering leading questions was very common. The process of cognitively emptying had "a direct tendency to make the scholar miserably superficial" (p. 83). "For why should he study if he knows from constant experiences that the teacher, by a leading question, will relieve him from all embarrassment?" (p. 83).

Page's "more excellent way". Immediately following the above passage, Page (1850) went on to elaborate a "more excellent way" (p. 84). Drawing from Pestalozzi, he commented that nature taught us that learners would be best served by being taught to depend mainly on their own resources. Page emphasised, though, that it was the teacher's responsibility to check thoroughly whether pupils were learning with *understanding*. It was appropriate for the teacher to give a word of suggestion, and to offer the scholar "a seasonable hint" for that could "save the scholar the needless loss of time". "The inquirer should never be frowned upon" (p. 84), and the "true way is neither to discourage inquiry nor answer the question" (p. 84). Page elaborated on his "more excellent way" but space does not permit this to be discussed in any detail here. Interestingly, Page's theories would have a large effect on education reforms in Japan and Taiwan in the 1880s and 1890s (Wing-kai To, 2011).

The Topaze Effect as a Re-invention of the Wheel

In developing his theory of didactical situations, Guy Brousseau (1997) linked the term "Topaze Effect" to the play *Topaze* by Marcel Pagnol (1928), in which a teacher (Topaze), wanting his student to learn something, led him to the correct answer by a series of leading questions—but, the students effectively learned nothing. The Topaze effect in mathematics education can be described in the following terms: When a teacher wants students to be able to answer a question but perceives that they are unable to obtain that answer by themselves, then the teacher, wishing to involve students in developing the answer without specifically "telling them", asks a series of leading questions aimed at moving the students towards the correct answer. The teacher has devised a way of thinking that she or he wants the students to adopt, and so the questions that the teacher asks are consistent with what the teacher wants the students to think and say. If the students answer a question inappropriately, then the teacher may ask further leading questions until an "appropriate" answer is obtained (Novotna & Hošpesova, 2007).

Most teachers of mathematics are aware of this phenomenon, even if they have never known it as *Topaze*. The leading questions asked by the teacher must be such that students can be expected to be able to answer them correctly. They are usually short, and require brief answers. They address the next small step in the total argument that is in the back of the teacher's mind. Often the Topaze effect is played out, apparently to everyone's satisfaction—the teacher asks cognitively low-level questions, and students answer them (often in chorus, if the teacher permits that). The students are happy when they are able to mouth the correct answers, and the teacher is happy when the students apparently reach the goals set for the lesson.

What is striking for us is how conceptually similar Brousseau's so-called "Topaze effect" is to Page's "drawing-out" process. Indeed, we cannot see any major difference, between the two. Both emphasise how teachers, in their desire to assist learners, are prone to ask sequences of questions that "empty" tasks of cognitive challenge. Although both Brousseau and Page emphasised how teachers did this, Page made the point well over a century before Brousseau did, and the cognitive emptying temptation to teachers was well recognised in North American education literature published in the second half of the 19th century (see, e.g., Cook, 1883; Hart, 1878). Almost certainly, this literature was not known by Brousseau (who made no reference to the writings of Page). Such a lack of knowledge of literature can contribute to a tendency to re-invent the wheel in mathematics education theory. Two of the authors of this paper have previously commented on this tendency at a MERGA conference (Ellerton & Clements, 2005).

Although it is usually assumed that the style of teaching characteristically associated with the term “Topaze effect” is seriously inadequate, and that the students learn little as a result of it, it could be the case that if the teacher has a strong understanding of mathematics, and some appreciation of the cognitive structures of her of his students with respect to the topic under consideration, then the students may be capable of identifying what is in their teacher’s minds and actually learn important methods for tackling problems. In other words, it should not be assumed that a question-answer style of teaching, in which a sequence of low-level questions is asked, is necessarily a poor pedagogical strategy for all teachers and for all students.

Mixed-Method Research is Needed on the Effects of Cognitive Emptying

Almost all mathematics teachers and mathematics educators are aware that the cognitive emptying phenomenon is to be found in many mathematics classrooms. It is also to be found in many interviews that researchers conduct when they seek to uncover the ways students are thinking (about certain concepts, or about problems, etc.). It is also true that often the teachers and the researchers are not aware that others would regard the discourse patterns which they control are examples of cognitive emptying. As Page (1850) observed, it is often almost impossible for a casual observer, or occasional analyst, to identify which components of a discourse feature cognitive emptying and which do not.

Most of the data offered by researchers and scholars in relation to the effects of cognitive emptying (or, equivalently, the Topaze Effect) have been in the form of classroom or interview transcripts—not unlike the transcripts already reported from Page (1850) in this paper. Such transcripts are highly suggestive of the kind of learning that generally is the result of teaching, or interviewing, in which cognitive emptying obviously features strongly. But, often, discourse from mathematics classrooms, or from research interviews, feature cognitive emptying in much more subtle ways than the occasional observer might imagine. Furthermore, the effects of such discourse should not be assumed to be the same on all learners. What is “cognitive emptying” for one learner may not be for another.

We would maintain that there is a need for careful *mixed-methods* research identifying the effects of standard mathematics classroom teaching sequences, and research interview sequences, on learners or interviewees. Undoubtedly, the reporting of transcripts is important, but so too are the quantitative reporting and analyses of pre-teaching (or pre-interview), post-teaching (or post-interview), and retention data. One without the other must convey an incomplete picture. Space does not permit us to elaborate the point here, but the following example might be useful. Vaiyavutjamai (2004) studied the discourse patterns in six ninth-grade algebra classes in Thailand, and these are summarised in Figure 1, which shows 16 graphs generated by analyses of the questions asked by teachers in 16 lessons. The vertical axes are labeled 0, 1, and 2, corresponding to whether a question by a teacher was classified as (a) procedural, or (b) low-level, or (c) high-level. The horizontal axes show the number of questions asked during the lesson. Decisions on the classifications of the levels of the questions were made after these terms had been carefully defined, and after discussion of classifications between Vaiyavutjamai and another independent researcher.

Linear Equations Lesson	Linear Inequations Lesson	Quadratic Equations Lesson

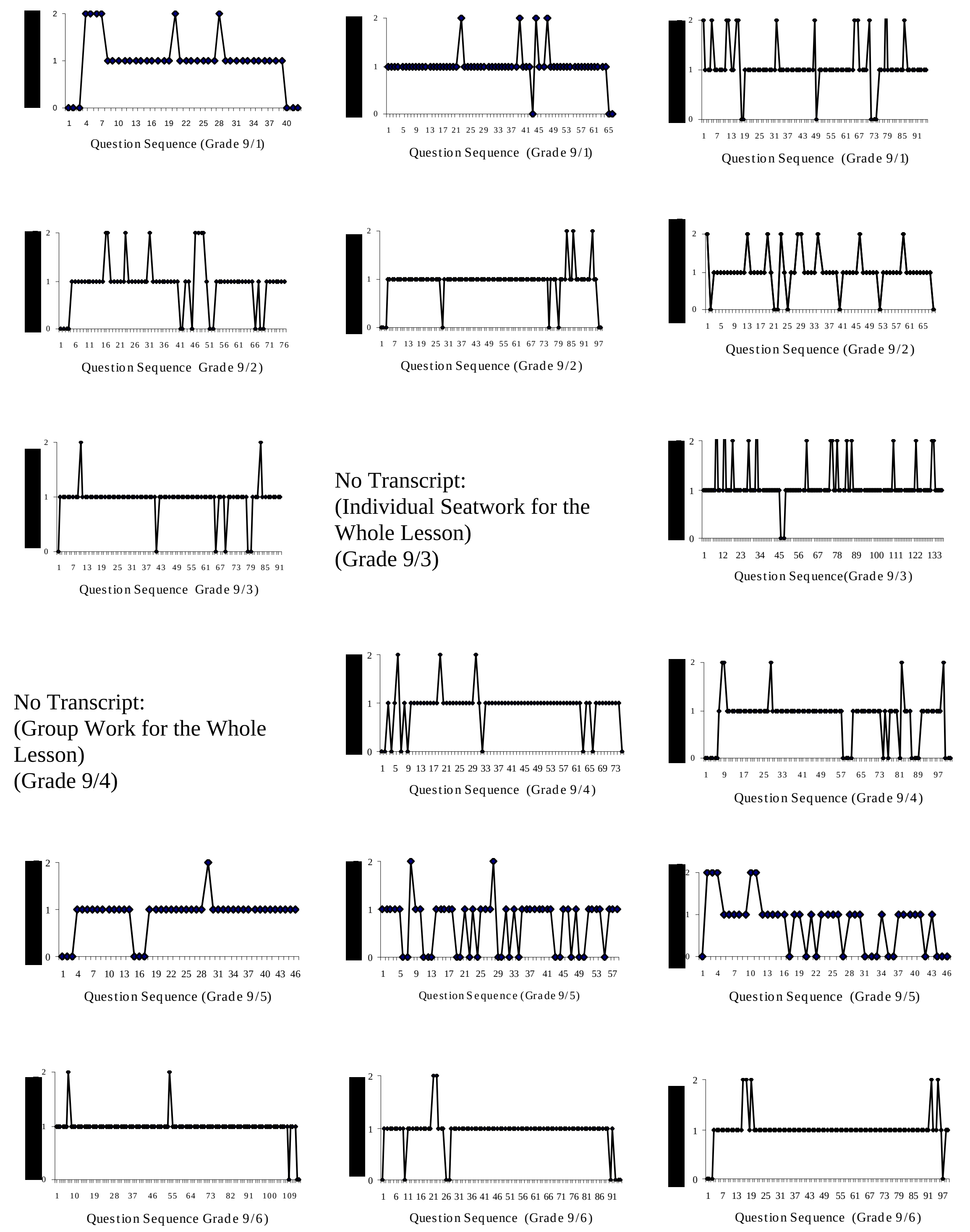

Figure 1. Teachers' patterns of questions asked in six algebra classes (Vaiyavutjamai, 2004).

These classes were in two schools (three classes in each school), and altogether four teachers were involved—whereas one teacher (Teacher *A*) taught all three classes in one school (these are labelled 9/1, 9/2 and 9/3 in Figure 1), three different teachers (Teachers *B, C, D*) taught each of the three classes (9/4, 9/5 and 9/6) in the other school. Class 9/1 and 9/4 were high-stream classes, and the other four classes were regarded as medium/low-stream classes. Vaiyavutjamai observed and audiotaped all 16 lessons, each of which was concerned with linear equations, or linear or quadratic inequalities, or quadratic equations. Since the graphs are small an impression of continuity is conveyed. In fact, however, the graphs comprise sets of discrete points (with adjoining points being joined by line intervals to assist interpretation). It can be seen from Figure 1 that each teacher tended to follow up a high-level question with a set of low-level questions.

When teaching linear equations and inequations to each of his three Grade 9 classes, Teacher *A* asked mainly low-level questions. In his lessons on quadratic equations he asked more high-level questions than the other three teachers did in their lessons on quadratic equations. Yet, even in his lessons on quadratic equations, Teacher *A* asked many more low- than high-level questions. Most of the questions asked by the three teachers at the second school were low-level, and Teacher *C* tended to ask more procedural questions than the other three participating teachers.

Vaiyavutjamai's (2004) analyses of the classroom transcripts revealed that all four participating teachers idiosyncratically combined elicitation, exposition and modelling in almost every lesson they taught. Typically, they would ask their classes a high-level question and then use low-level questions to channel their students' thinking toward an answer that they deemed to be acceptable. This elicitation strategy seemed to empty the high-level tasks of cognitive challenge. If teachers needed to teach something quickly (for example, to complete a lesson, or to finish a topic before a test), or if they thought the content was difficult, they tended to move into "expository" mode. During seatwork, students answered exercises by imitating the approaches in the model examples.

Figure 2 shows pre-teaching, post-teaching and retention data on strictly parallel tests for the six classes. It is interesting that the apparently cognitive-emptying discourse patterns in each class generated improvement, even at the retention stage, with both top-stream classes, but with the other classes, the retention mean performances differed by only small amounts from the pre-teaching mean performances.

Concluding Comments

Theories such as the "Topaze Effect" (and the "Jourdain Effect" and the "Dienes Effect", both of which Brousseau (1997) also described) are not theories unless there is the possibility that they can be refuted. In this paper our brief description of Vaiyavutjamai's (2004) study has suggested that both qualitative and quantitative research methods should be combined before such theories can be substantiated.

The paper also suggested that mathematics education researchers need to develop a stronger knowledge of our own history—including knowledge of theories used in earlier times. We cannot afford to keep on re-inventing the wheel.

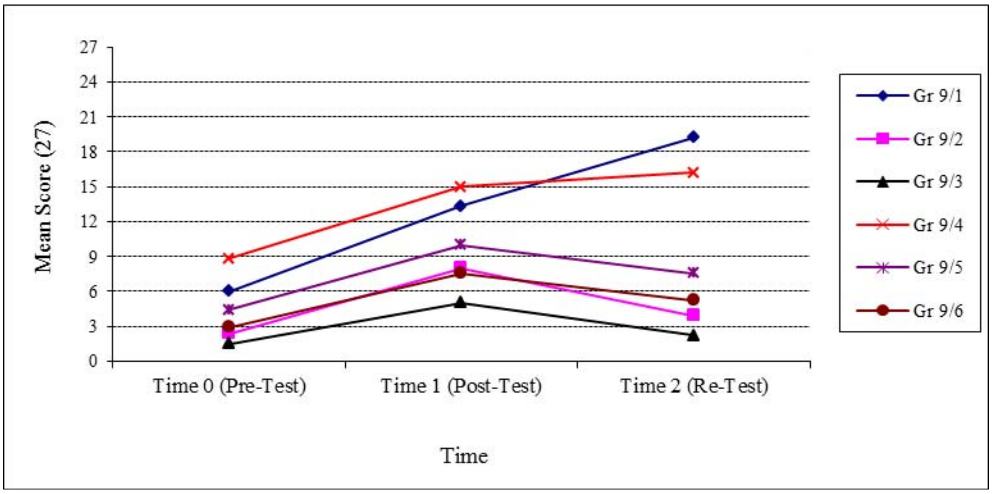

Figure 2. Pre- and post-teaching mean scores of six classes (Vaiyavutjamai, 2004).

References

Bidwell, J. K., & Clason, R. G. (Eds.). (1970). *Readings in the history of mathematics education.* Washington, D.C.: National Council of Teachers of Mathematics.

Brousseau, G. (1997). *Theory of didactical situations in mathematics*, Dordrecht, The Netherlands: Kluwer Academic Publishers.

Colburn, W. (1821). *Arithmetic on the plan of Pestalozzi.* Boston, MA: Cummings & Hilliard.

Colburn, W. (1822*a*). *First lessons in arithmetic on the plan of Pestalozzi.* Boston, MA: Cummings & Hilliard.

Colburn, W. (1822*b*). *Arithmetic, being a sequel to the First Lessons in Arithmetic.* Boston, MA: Cummings & Hilliard.

Cook, J. W. (1883). *Methods in written arithmetic.* Chicago: A. Flanagan Co., Publishers

Ellerton, N. F., & Clements, M. A. (2005). A mathematics education ghost story: Herbartianism and school mathematics. In P. Clarkson, A. Downton, D. Gronn, M. Horne, A. McDonagh, R. Pierce & A. Roche (Eds.), *Building connections: Research, theory and practice* (pp. 313-320). Sydney, Australia: Mathematics Education Research Group of Australasia.

Hart, J. S. (1878). *In the school-room: Or, chapters in the philosophy of education.* Philadelphia, PA: Eldredge & Brother.

Novotna, J., & Hošpesova, A. (2007). What is the price of Topaze? In J.-H. Woo, H.-C. Lew, K.-S. Park, & D.-Y. Seo (Eds.), *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 25–32). Seoul, Korea: International Group for the Psychology of Mathematics Education.

Page, D. P. (1850). *Theory and practice of teaching: The motives and methods of good school-keeping* (12th ed.). New York, NY: A. S. Barnes & Co.

Pagnol, M. (1958). *Topaze* (trans. R. Waldinger). Great Neck, NY: Barron's Educational Series.

Soëtard, M. (1981). *Pestalozzi ou la naissance de l'éducateur. Étude sur l'évolution de la pensée et de l'action du pédagogue suisse (1746–1827)*, Berne, Switzerland: Publications Universitaires Européennes.

Soëtard, M. (1994). Johann Heinrich Pestalozzi (1746–1827). *Prospects: The Quarterly Review of Comparative Education, 24*(1/2), 297–310.

Vaiyavutjamai, P. (2004). The effects of teaching on Form 3 students' learning of equations and inequations. In I. P. Cheong, H. S. Harkirat, I. J. Kyeleve, & O. Chukwu (Eds.), *Globalisation trends in science, mathematics, and technical education* (pp. 280–289). Gadong: Universiti Brunei Darussalam.

Wing-kai To (2011). Bridgewater Normal School and Isawa Shuji's reforms of modern teacher education in Meiji Japan. *Journal of East Asian Cultural Interaction Studies*, *2*, 67–86.